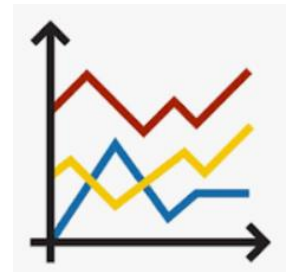

THE EQUATION OF A LINE –

$$y - y_1 = m(x - x_1)$$

The slope-intercept form of a line, $y = mx + b$, is perfect when you have the slope and the y-intercept. But the odds of this are slim. It's much more likely that you'll be working with the slope and some point on the line other than the y-intercept. So, in the following formula, m is the slope (as before), while (x_1, y_1) represents any given point on the line.



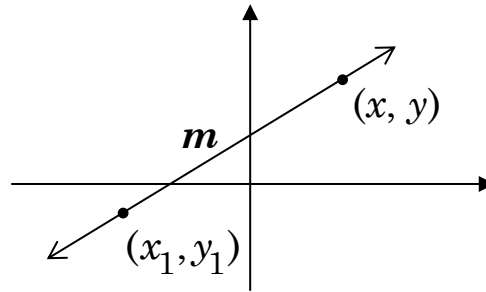
□ THE POINT-SLOPE FORMULA

THEOREM: The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

**The Point-Slope
Form of a Line**

PROOF: We begin by sketching the line and labeling the given point (x_1, y_1) , the slope m , and a *generic point* (x, y) :



On the one hand, the slope of the line is given by m .

On the other hand, the slope of the line can be calculated using the two points (x, y) and (x_1, y_1) : $\frac{y - y_1}{x - x_1}$. And, of course, these

two slopes must be the same (since there's only one line involved):

$$\frac{y - y_1}{x - x_1} = m$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

This is the form of a line that is used in Calculus.

❑ **BONUS DERIVATION**

Using the point-slope form of a line above, we can derive the slope-intercept form of a line: $y = mx + b$. Here's how:

We assume that the slope of a line is given by m , and that the y -intercept is $(0, b)$, which is simply a point on the line; so it's the point (x_1, y_1) in the formula $y - y_1 = m(x - x_1)$. Thus,

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - b = m(x - 0)$$

$$\Rightarrow y - b = mx$$

$$\Rightarrow y = mx + b \quad \text{and we're done!}$$

The Equation of a Line – $y - y_1 = m(x - x_1)$

□ EXAMPLES OF THE POINT-SLOPE FORMULA

EXAMPLE 1: Find the equation of the line whose slope is -3 and which passes through the point $(8, -2)$.

Solution: This is precisely the data we need to use the point-slope form: $y - y_1 = m(x - x_1)$. We're given the slope, so $m = -3$. We're also given a point on the line, so $(x_1, y_1) = (8, -2)$. Plugging these values into the point-slope form gives us

$$y - (-2) = -3(x - 8), \text{ or}$$

$$y + 2 = -3(x - 8)$$

EXAMPLE 2: Find the equation of the line passing through the two points $(3, -5)$ and $(-2, -8)$.

Solution: The point-slope form, $y - y_1 = m(x - x_1)$, requires a point (we have two of them), and the slope, which we'll have to calculate ourselves:

$$m = \frac{\Delta y}{\Delta x} = \frac{-5 - (-8)}{3 - (-2)} = \frac{-5 + 8}{3 + 2} = \frac{3}{5}$$

Now, using the point $(3, -5)$ (either point would work), we get our equation

$$y - (-5) = \frac{3}{5}(x - 3), \text{ or } y + 5 = \frac{3}{5}(x - 3)$$

Homework

1. Use the point-slope formula to find the equation of the line with slope 7 and passing through the point $(6, -8)$.

$$\text{The Equation of a Line} - y - y_1 = m(x - x_1)$$

2. Use the point-slope formula to find the equation of the line with slope 0 and passing through the point $(-17, 9)$.
3. Use the point-slope formula to find the equation of the line with slope $-\frac{4}{7}$ and passing through the point $(\frac{1}{2}, \pi)$.
4. Use the point-slope formula to find the equation of the line which passes through the points $(-2, 4)$ and $(5, -5)$.
5. Use the point-slope formula to find the equation of the line which passes through the points $(\pi, \sqrt{2})$ and $(-3, 1)$.

Solutions

1. $y + 8 = 7(x - 6)$

2. $y - 9 = 0$

3. $y - \pi = -\frac{4}{7}\left(x - \frac{1}{2}\right)$

4. $y - 4 = -\frac{9}{7}(x + 2)$

5. $y - \sqrt{2} = \frac{1 - \sqrt{2}}{-3 - \pi}(x - \pi)$ The slope can also be written: $\frac{\sqrt{2} - 1}{\pi + 3}$.

*An investment in knowledge
always pays the best interest.*

Benjamin Franklin